Towards a QCD-based Description of Non-Leptonic Multibody B Decays

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Contents

Introduction

Levels of complexity in B decays

- Purely leptonic f_B
- Inclusive semileptonic: Heavy Quark Expansion (HQE)
- Inclusive Nonleptonic (Lifetimes, Mixing): HQE
- Exclusive semileptonic: $F^{B\to M}(q^2)$
- Inclusive FCNC $b \to s\ell\ell$ and $b \to s\gamma$: (HQE + ...)
- Exclusive FCNC $b \to s\ell\ell$ and $b \to s\gamma$: $F^{B\to M}(q^2)$ + ...
- Two-Body Non-leptonic: QCD Factorization (QCD-F)
- Multi-Body Non-Leptonic: ???

$$\boxed{\text{Make Use of the fact that } \alpha_s(m_b) \ll 1}$$

Standard theoretical machinery

- Effective Hamiltonian

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_i C_i(\mu) O_i(\mu)$$

- C_i: Wilson Coefficients: short distance, $\alpha_s(M_W)$
- O_i: Local operators: Long distance physics
- μ: renormalization point
- Decay amplitudes:

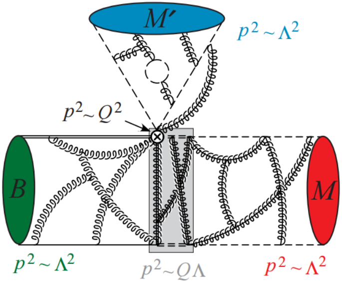
$$\mathcal{A}(B \to f) = \frac{G_F}{\sqrt{2}} \sum_i C_i(\mu) \langle f | O_i(\mu) | B \rangle$$

- How to compute the operator matrix elements?

Factorization

- $\langle f|O_i(\mu)|B\rangle$ still contains the large scale m_b
- There are contributiions which can be calculated perturbatively: $\alpha_s(m_b)$
- Factorization of these perturbative contributions
- Suitable definition of (universal) non-perturbative quantities
- OPE and Effective Field Theories

Factorization in two-body non-leptonics

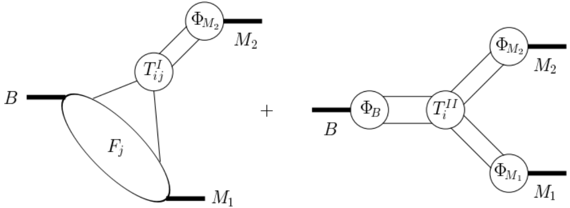


$$\langle M_{\bar n} M_n | O_i | B \rangle = \langle M_{\bar n} | \bar h_v \Gamma \xi_{\bar n} | B \rangle$$

$$\times \int dz\, T_i(z) \langle M_n | \bar\chi_n(zn) \Gamma' \chi_n(0) | 0 \rangle$$

$$\sim F^{B \to M}\, T_i \otimes \phi_M$$

(Beneke, Buchalla, Neubert, Sachrajda, Bauer, Pirjol, Rothstein, Stewart, ...)



- Established methodology for two body decays
- Anatomy of $B \to D\pi$ and $B \to \pi\pi$ is understood
- Phenomenology works
- Indications of the presence of subleading terms
- ... but the two body decays are only a small fraction of the total non-leptonic width!
- Clear need for a QCD-based description of multi-body decays

Three-body non-leptonics

Kinematics: $p_B \to p_1 + p_2 + p_3$

- Two independent kinematical variables $p_i^2 = 0$

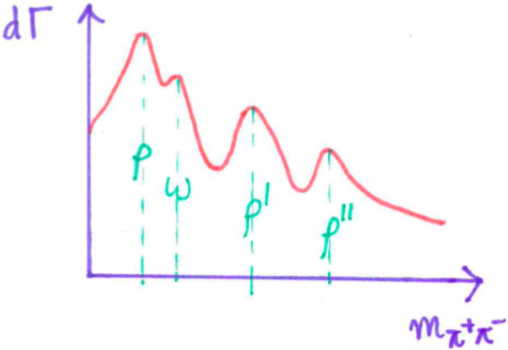
$$s_{ij}^2 = (p_i + p_j)^2 \quad s_{12} + s_{13} + s_{23} = M_B^2$$

Historically:

- "Isobar" Model:
- Description via pseudo two-particle decays:

$$(B \to M\, M_1 M_2) = (B \to M^* M \quad \text{and} \quad M^* \to M_1 M_2)$$

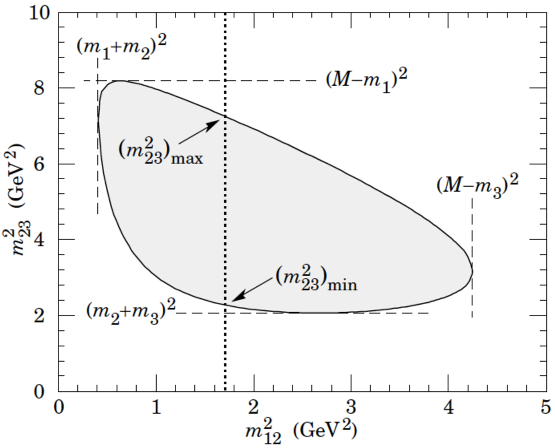
- sum over all possibilities for M^*, including $\Gamma(M^*)$
- possibly add a flat "non-resonant" backgound!



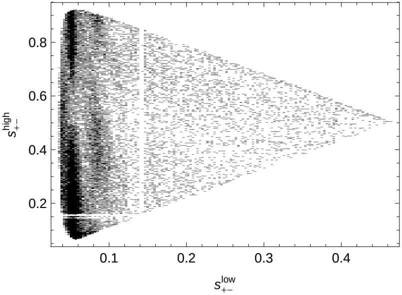
(sketch borrowed from J. Virto)

$B \to \pi\pi\pi$

Study the Dalitz Distribution:



Specifically for $B^+ \to \pi^+\pi^-\pi^+$



(Plot form LHCb arXiv:1408.5373)

Dalitz Plot is symmetric:

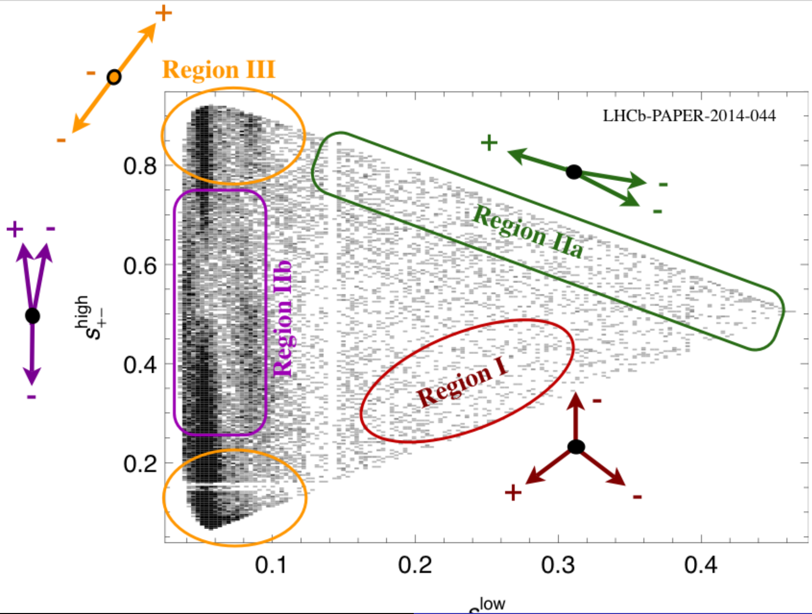
$s_{12} = s_{+-}^{\rm low} \quad s_{23} = s_{+-}^{\rm high}$

$s_{12} = s_{++}$

Regions

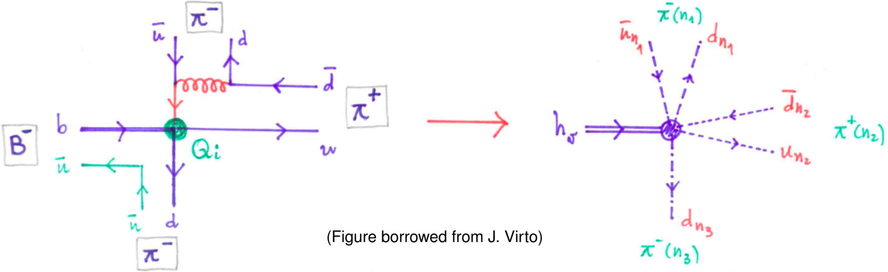
Split the Dalitz Plot into Regions:

- **Region 1:** "Mercedes Star"
  $s_{++} \sim s_{+-}^{\rm low} \sim s_{+-}^{\rm high} \sim 1/3$
- **Region 2:** Collinear Decay Products
  - **Region 2a:** $(\pi^+\pi^+)_{\rm coll}$ recoil against π^-
    $s_{++} \sim 0, \quad s_{+-}^{\rm low} \sim s_{+-}^{\rm high} \sim 1/2$
  - **Region 2b:** $(\pi^+\pi^-)_{\rm coll}$ recoil against π^+
    $s_{+-}^{\rm low} \sim 0, \quad s_{++} \sim s_{+-}^{\rm high} \sim 1/2$
- **Region 3:** Soft Decay Products
  - **Region 3a:** Soft π^+
    $s_{++} \sim s_{+-}^{\rm low} \sim 0 \quad s_{+-}^{\rm high} \sim 1$
  - **Region 3b:** Soft π^-
    $s_{+-}^{\rm low} \sim_{+-}^{\rm high} \sim 0, \quad s_{++} \sim 1$


Region III
Region IIb
Region IIa
Region I
LHCb-PAPER-2014-044
0.8
0.6
0.4
0.2
0.1
0.2
0.3
0.4
s_{+-}^{high}
s^{low}
+
-

Region 1: The Center

- Three "disconnected" collinear directions: n_1 n_2 n_3



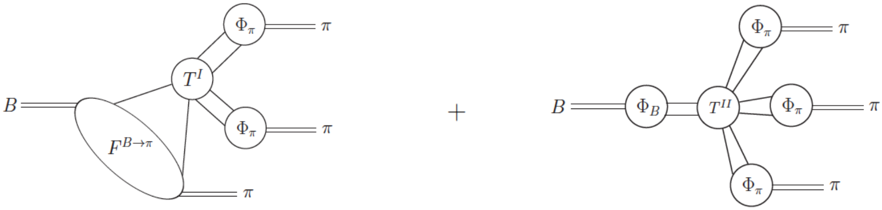
(Figure borrowed from J. Virto)

$$\langle \pi^-_{n_1} \pi^+_{n_2} \pi^-_{n_3} | O_i | B \rangle = \langle \pi^-_{n_3} | \bar{d}_{n_3} \Gamma_3 h_v | B \rangle$$

$$\times \int dudv\, T_i(u,v) \langle \pi^-_{n_1} | \bar{d}_{n_1}(\bar{u}) \Gamma_1 u_{n_1}(u) | 0 \rangle \langle \pi^+_{n_2} | \bar{u}_{n_2}(\bar{v}) \Gamma_2 d_{n_2}(v) | 0 \rangle$$

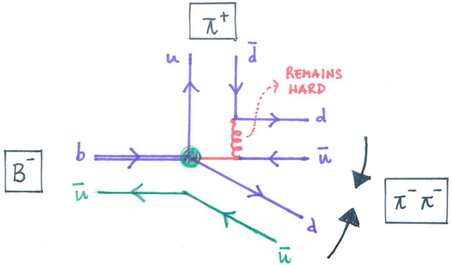
$$\sim F^{B\to\pi}\, T_i \otimes \phi_\pi \otimes \phi_\pi$$

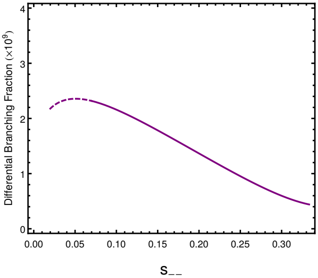
- $1/m_b^2$ and α_s supressed with repect to a two body decay
- At leading order / leading power / leading twist all convolutions are finite
  $\rightarrow$ factorization:



Extrapolation to collinear $\pi^-\pi^-$

- There are no resonances in this channel
- No infrared / collinear problems expected
- Perturbative result expected to be regular: No "soft" propagators

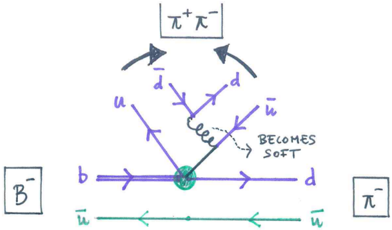


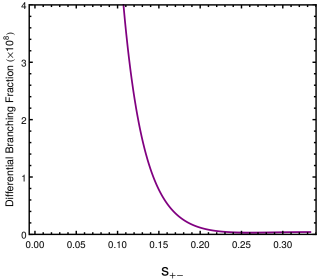


$$\frac{d\Gamma}{ds_{--}\,ds_{+-}} \simeq 0.84\,\Gamma_0\, f_+(m_B^2/2)^2 + \mathcal{O}(s_{--})$$

Extrapolation to collinear $\pi^+\pi^-$

- There are resonances in this channel: ρ. ω, ...
- Perturbative result expected to be IR singular
- “soft” propagators

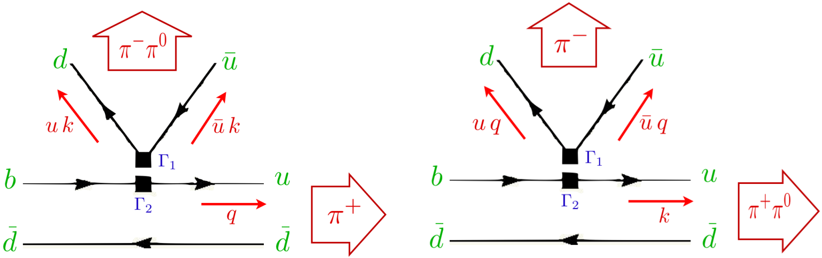




$$\frac{d\Gamma}{ds_{+-}\,ds_{--}} \simeq \frac{0.38}{s_{+-}}\,\Gamma_0\, f_+(0)^2 + \text{regular}$$

Region 2b: new non-perturbative input

- Factorization breaks down in the resonance regions
- New, nonperturbative input is needed
- Three-body decay resembles two-body decay



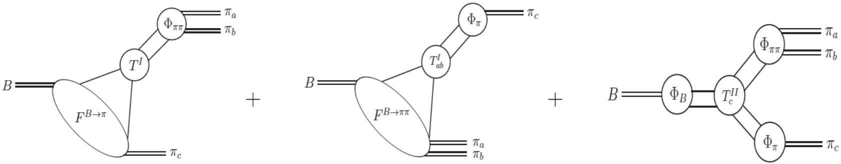
- Operators are the same as in two-body decays ...

- ... but the final states are different

$$
\begin{aligned}
&\langle \pi^-_{\bar n} \pi^+_{\bar n} \pi^-_n | O_i | B \rangle = \\
&\langle \pi^-_n | \bar h_v \Gamma \xi_n | B \rangle \times \int dz\, T_1(z) \langle \pi^-_{\bar n} \pi^+_{\bar n} | \bar\chi_{\bar n}(z\bar n) \Gamma' \chi_{\bar n}(0) | 0 \rangle \\
&+ \langle \pi^-_{\bar n} \pi^+_{\bar n} | \bar h_v \Gamma \xi_{\bar n} | B \rangle \times \int dz\, T_2(z) \langle \pi^-_n | \bar\chi_{\bar n}(zn) \Gamma' \chi_n(0) | 0 \rangle \\
&\sim F^{B\to\pi}\, T_1 \otimes \phi_{\pi\pi} + F^{B\to\pi\pi}\, T_2 \otimes \phi_\pi
\end{aligned}
$$

- Two-Pion light-cone distribution (Polyakov, Diehl, Gousset, ...)
- Generalized (soft) Form factor (Feldmann, Khofjamirian, van Dyk, ThM ...)

- Factorization formula similar to the two-body case



Two-Pion Light Cone Distribution

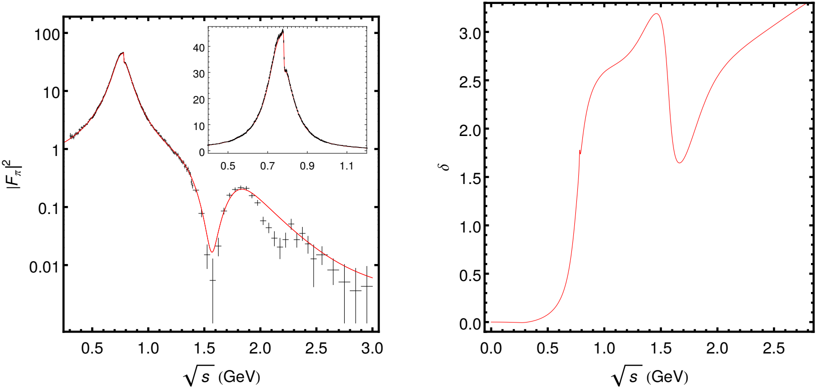
- Definition: $s = (k_1 + k_2)^2$, $k_1 = \zeta k_{12}$, $k_2 = \bar{\zeta} k_{12}$]

$$\phi^q_{\pi\pi}(z, \zeta, s) = \int \frac{dx^-}{2\pi} e^{iz(k_{12}^+ x^-)} \langle \pi^+(k_1)\pi^-(k_2)|\bar{q}(x^- n_-)\not{n}_+ q(0)|0\rangle$$

- Normalization from the local limit:

$$\int dz\, \phi_{\pi\pi}(z, \zeta, s) = (2\zeta - 1) F_\pi(s) \quad \text{(pion time-like FF)}$$

- $F_\pi(s)$: Data (BaBar) + Theory (χPT, Asymptotics...)
- z and ζ dependence asymptotically known



(Hanhart, Kubis, ...)

Timelike Pion Form Factor known from Data

Generalized (soft) Form factor

- Relevant Form factor:

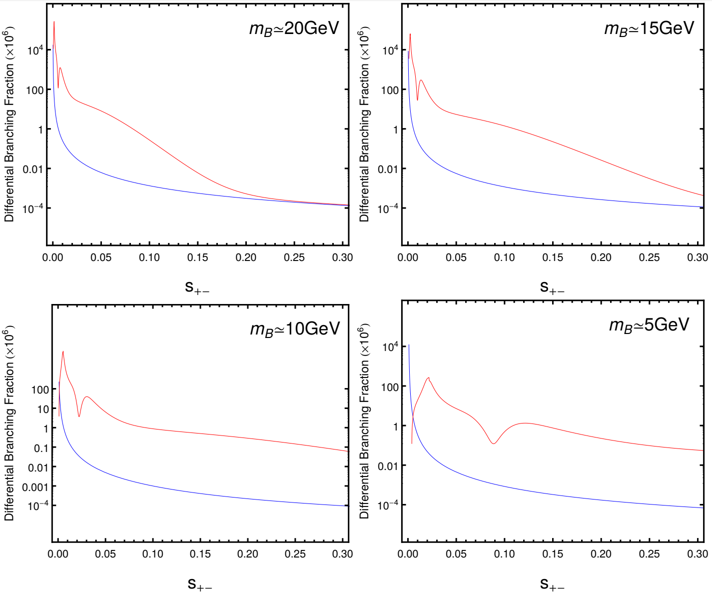
$$\langle \pi^+(k_1)\pi^-(k_2)|\,\bar{u}\not{k}_3 P_{L,R} b\,|B^-(p)\rangle = \mp \frac{m_\pi}{2} F_t(\zeta, s)$$

- $F_t(\zeta, s)$ can be related to the two-pion light-cone distribution via a Light-Cone Sum Rule (Khodjamirian, Hambrock)

$$F_t(\zeta, s) = \frac{m_b^2}{\sqrt{2}\hat{f}_B m_\pi} \int_{u_0}^1 \frac{du}{u} \exp\left[\frac{(1+s\bar{u})m_B^2}{M^2} - \frac{m_b^2}{uM^2}\right] \phi_{\pi\pi}(u, \zeta, s)$$

Merging the Regions ...

- The starting point is the large-m_b limit
- Do the regions match properly?
- Is m_b large enough?
- Is there a central region for $m_b \sim 5$ GeV?


$m_B \simeq 20$GeV
$m_B \simeq 15$GeV
$m_B \simeq 10$GeV
$m_B \simeq 5$GeV
Differential Branching Fraction $(\times 10^6)$
10^4
100
10
1
0.1
0.01
0.001
10^{-4}
0.00
0.05
0.10
0.15
0.20
0.25
0.30
s_{+-}

- Probably there is no perturbatively calculable central region for realistic m_b
- For realistic m_b the Dalitz plot consists only of edges
- Three-body decays become quasi two-body
- The factorzation formula is the one known form the two-body decays with new non-perturbative input
- These are just first results, many more checks need to be performed.

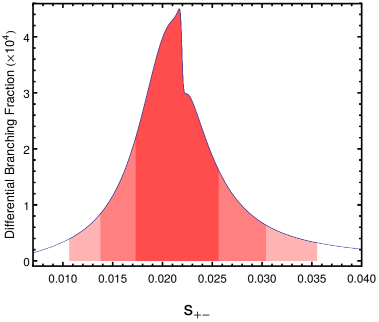
First Application: $B \to \rho\pi$

- Amplitude near $s_{+-} \ll m_b^2$

$$\mathcal{A} \sim \frac{G_F}{\sqrt{2}} \Big[4 m_B^2 f_0(s_{+-})(2\zeta - 1) F_\pi(s_{+-})(a_2 + a_4) + f_\pi m_\pi (a_1 - a_4) F_t(\zeta, s_{+-}) \Big]$$

- Definition of the ρ: Integration around the ρ mass:

$$BR(B^- \to \rho\,\pi^-) \simeq \int_0^1 ds_{++} \int_{s_\rho^-}^{s_\rho^+} ds_{+-} \, \frac{\tau_B \, m_B |\mathcal{A}|^2}{32(2\pi)^3}$$



with $s_\rho^\pm = (m_\rho \pm n\Gamma_\rho)^2/m_B^2$

$$BR(B^+ \to \rho\pi^+) \simeq 9.4 \cdot 10^{-6} \quad (n = 0.5)$$
$$BR(B^+ \to \rho\pi^+) \simeq 12.8 \cdot 10^{-6} \quad (n = 1)$$
$$BR(B^+ \to \rho\pi^+) \simeq 14.1 \cdot 10^{-6} \quad (n = 1.5)$$

$$BR(B^+ \to \rho\pi^+)_{\mathrm{EXP}} = (8.3 \pm 1.2) \cdot 10^{-6}$$
$$BR(B^+ \to \rho\pi^+)_{\mathrm{QCDF}} = (11.9^{+7.8}_{-6.1}) \cdot 10^{-6}$$

CP Violation Studies

- Three-Body Decays contain more information than two-body:
- Compare the Dalitz Plots of B^+ vs. B^-
- Bin-wise asymmetry

$$\Delta(i) = \frac{N(i) - \bar{N}(i)}{N(i) + \bar{N}(i)}$$

- Miranda Procedure: Use the significance (well suited for “noisy” data)

$$S_{\mathrm{CP}}(i) = \frac{N(i) - \bar{N}(i)}{\sqrt{N(i) + \bar{N}(i)}}$$

= more robust probe of CP violation

(Bediga, Bigi, et al. ...)

- CP Violation is distributed over the Dalitz plot
- Strong phases depend on the bin:
  eg. Breit Wigner

$$\mathrm{Im}\, BW(s) = \frac{m_R \Gamma_R}{(m_R^2 - s)^2 + m_R^2 \Gamma_R^2}$$

- Expample: $B \to K\pi\pi$:
  Interferences between different channels
  e.g $B \to K^*\pi \to K\pi\pi$ and $B \to K\rho \to K\pi\pi$

Phys. Rev. D 90, 112004 (2014)

Recent LHCb Results

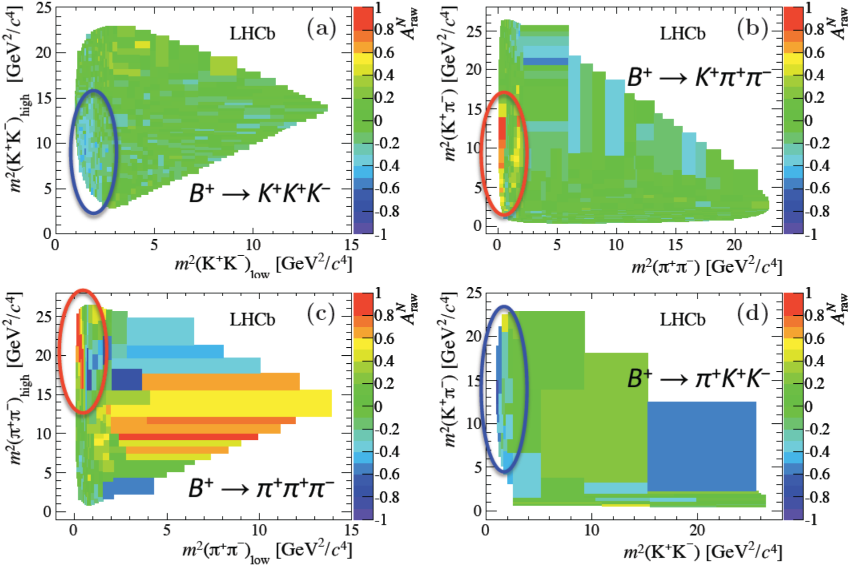
- LHCb analysis of $B^+ \to h^+ h^- h'^+$ decays using full LHC Run 1 data sample
- Much larger samples (20-40x) than *B*-factories
- CP asymmetries measured in full phase space:

$$A_{CP}(B^\pm \to K^\pm \pi^+ \pi^-) = +0.025 \pm 0.004 \pm 0.004 \pm 0.007,$$

$$A_{CP}(B^\pm \to K^\pm K^+ K^-) = -0.036 \pm 0.004 \pm 0.002 \pm 0.007,$$

$$A_{CP}(B^\pm \to \pi^\pm \pi^+ \pi^-) = +0.058 \pm 0.008 \pm 0.009 \pm 0.007,$$

$$A_{CP}(B^\pm \to \pi^\pm K^+ K^-) = -0.123 \pm 0.017 \pm 0.012 \pm 0.007,$$


LHCb
(a)
$B^+ \to K^+K^+K^-$
$m^2(K^+K^-)_{high}$ [GeV$^2/c^4$]
$m^2(K^+K^-)_{low}$ [GeV$^2/c^4$]
A^N_{raw}
LHCb
(b)
$B^+ \to K^+\pi^+\pi^-$
$m^2(K^+\pi^-)$ [GeV$^2/c^4$]
$m^2(\pi^+\pi^-)$ [GeV$^2/c^4$]
A^N_{raw}
LHCb
(c)
$B^+ \to \pi^+\pi^+\pi^-$
$m^2(\pi^+\pi^-)_{high}$ [GeV$^2/c^4$]
$m^2(\pi^+\pi^-)_{low}$ [GeV$^2/c^4$]
A^N_{raw}
LHCb
(d)
$B^+ \to \pi^+K^+K^-$
$m^2(K^+\pi^-)$ [GeV$^2/c^4$]
$m^2(K^+K^-)$ [GeV$^2/c^4$]
A^N_{raw}

- Huge CP Asymmetries in some regions of phase space
- Needs a full amplitude analysis, including the phases
- Experimental input needed (like the time like pion form factor)
- QCD Factorization may provided a tool which eventually may be better than what was done for the two body decays!

Summary

- Multi-body decays ...
  - ... are abundant
  - ... contain important information in their kinematic distributions
  - ... are theoretically most complex
- QCD based Ansatz: QCD factorization
- No perturbative central region, even for B decays
- Quasi two-body with new non-perturbative input

More work needed to establish QCD-Factoriztion!